1. For a three-node bar element with coordinates(x1,y1) = (1,1),(x2,y2) =(3,1)and(x3,y3) = (2,2), determine an expression for its orientationas a function of the local coordinateξ. Find the orientation vector atnode 3.

n = 3;

xi = linspace (-1, 1, n);

C = [1 1; 3 1; 2 2];

for i = 1:n

J(:, :, i) = C'\* lin\_deriv (xi(i));

Jacobian(:, :, i) = norm (J(:, :, i));

r(:, :, i) = J(:, :, i)/Jacobian(:, :, i);

end

r(:, :, 2)';

function dn = lin\_deriv (xi)

dn = [xi - 1/2

xi + 1/2

-2\*xi];

end



2. Using integration, calculate the length of the element in the last exer-cise.

syms xi

C = [1 1; 3 1; 2 2];

J = C'\* lin\_deriv (xi);

eqs = norm(J);

int(eqs, -1, 1)

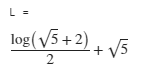
function dn = lin\_deriv (xi)

dn = [xi - 1/2

xi + 1/2

-2\*xi];

end



3. Study the limitations for the location of intermediate nodes in a four-node bar element. Assume that the nodes are placed simmetrically.

syms xi alpha L

C = [-L/2 ; L/2 ; -L/6; alpha\*L];

j = C'\* lin\_deriv (xi);

simplify(j);

J = subs(j, L, 1) == 0;

alfa = [subs(J, xi, -1/3); ...

subs(J, xi, 1 )];

for i = 1:2

A(i, 1) = solve (alfa(i));

end

fprintf ('[%0.2f , %0.2f]\*L\n',A)

function dn = lin\_deriv (xi)

n = [1.0/16.0\*( -9.0\*xi^3 + 9.0\*xi\*xi + xi - 1.0)

1.0/16.0\*( 9.0\*xi^3 + 9.0\*xi\*xi - xi - 1.0)

1.0/16.0\*( 27.0\*xi^3 - 9.0\*xi\*xi - 27.0\*xi + 9.0)

1.0/16.0\*(-27.0\*xi^3 - 9.0\*xi\*xi + 27.0\*xi + 9.0)];

dn = diff(n);

end



4. By inspection determine the Jacobian norm of the four-node elementshown below. Then, compare the result with the one obtained by usingthe Jacobian matrix.

clear all

clc

syms xi eta

n = 2;

x = linspace (-1, 1, n);

C = [0 0 0; 1 0 0; 1 1 1; 0 1 1];

dn = quad4\_deriv (xi, eta);

J1 = C' \* subs (dn, [xi eta], [x(1) x(1)]);

J2 = C' \* subs (dn, [xi eta], [x(2) x(1)]);

J3 = C' \* subs (dn, [xi eta], [x(2) x(2)]);

J4 = C' \* subs (dn, [xi eta], [x(1) x(2)]);

NormJ1 = sqrt((det(J1([1 2], [1 2])))^2 + ...

(det(J1([2 3], [1 2])))^2 + ...

(det(J1([3 1], [1 2])))^2);

NormJ2 = sqrt((det(J2([1 2], [1 2]))^2) + ...

(det(J2([2 3], [1 2]))^2) + ...

(det(J2([3 1], [1 2]))^2));

NormJ3 = sqrt((det(J3([1 2], [1 2]))^2) + ...

(det(J3([2 3], [1 2]))^2) + ...

(det(J3([3 1], [1 2]))^2));

NormJ4 = sqrt((det(J4([1 2], [1 2]))^2) + ...

(det(J4([2 3], [1 2]))^2) + ...

(det(J4([3 1], [1 2]))^2));

function dn = quad4\_deriv (xi, eta)

n = [1.0/4.0 \* (1 - xi) \* (1 - eta)

1.0/4.0 \* (1 + xi) \* (1 - eta)

1.0/4.0 \* (1 + xi) \* (1 + eta)

1.0/4.0 \* (1 - xi) \* (1 + eta)];

dn = [diff(n, xi), diff(n, eta)];

end



5. Compute the Jacobian norm as function of ξ and η of the element in the last exercise changing the coordinates of node 3 to x3= (1,1,0).Later calculate the surface area by integration.

syms xi eta

n = 2;

x = linspace (-1, 1, n);

C = [0 0 0; 1 0 0; 1 1 0; 0 1 1];

dn = quad4\_deriv (xi, eta);

J = C' \* dn;

NormJ = sqrt((det(J([1 2], [1 2])))^2 + ...

(det(J([2 3], [1 2])))^2 + ...

(det(J([3 1], [1 2])))^2);

int(int(NormJ, xi, -1, 1), eta, -1, 1)

var = vpa(ans);

function dn = quad4\_deriv (xi, eta)

n = [1.0/4.0 \* (1 - xi) \* (1 - eta)

1.0/4.0 \* (1 + xi) \* (1 - eta)

1.0/4.0 \* (1 + xi) \* (1 + eta)

1.0/4.0 \* (1 - xi) \* (1 + eta)];

dn = [diff(n, xi), diff(n, eta)];

end

